Peculiarities in multichannel interaction amplitudes for meson-meson scattering and scalar meson spectroscopy

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Abstract

Interactions in coupled channels $\pi\pi$, $K\bar{K}$ and an effective $2\pi 2\pi$ in scalar-isoscalar wave have been analysed. Influence of interchannel couplings on analytical structure of multichannel interaction amplitudes has been studied. Interplay of S-matrix zeroes and poles and their relation with parameters of scalar resonances has been investigated.

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Structure and dynamics of scalar mesons is still not clear and is a subject of many theoretical and experimental efforts [1]. Their multichannel decays and possible interference with glueball states below 2 GeV make results of analyses very model-dependent. In our model we use separable potentials [2, 3] for three coupled channels: $\pi\pi$, $K\overline{K}$ and $\sigma\sigma$ (an effective $2\pi 2\pi$) in S-wave with isospin 0. We also do not assume the existence of any scalar mesons before fitting experimental data. As experimental data for the $\pi^+\pi^-$ interactions we use "down-flat" and "up-flat" solutions of [4] in the effective two-pion mass $m_{\pi\pi}$ range from 600 MeV to 1600 MeV. Below 600 MeV we use the $\pi\pi$ data described in [2] and the $K\overline{K}$ channel data of [5].

We solve a system of three Lippmann-Schwinger coupled equations and find the Jost functions $D(k_1, k_2, k_3)$. Then we construct S-matrix elements and express them in terms of phase shifts and inelasticities which we fit to available experimental data. In this way we determine 14 free parameters of our model, finding two solutions to the "up-flat" data and four solutions to the "down-flat" data. We obtain χ^2 values smaller than 116 for 102 points. An example of the energy dependence of the $\pi\pi$ phase shifts and inelasticities for one of our solutions is shown in Fig. 1.

In order to investigate a role played by the interchannel interactions in the scalar meson dynamics we first study the analytical structure of the S-matrix elements in the fully decoupled case where all the interchannel couplings are equal to zero. In such a case one can recognize in which channel particular poles are created. When the interchannel couplings are switched on, all the poles change positions and split into 2^{n-1} (n is the number of channels) poles lying on different sheets labeled by the signs of the imaginary parts of the complex momenta in various channels. Both in the fully decoupled and in the coupled cases all the diagonal S-matrix elements can be expressed as ratios of two Jost functions. The denominators of these matrix elements are the same but their numerators are different. Therefore the positions of poles are common for all the S-matrix elements but the positions of zeroes depend on the chosen channel. Only poles and zeroes which lie close enough to the physical region have a significant influence on the experimental phase shifts and inelasticities. We then study poles and zeroes nearby the physical regions and relate scalar resonances to them.

In Fig. 2 we show schematic positions of some poles and zeroes of the $S_{\pi\pi}$-matrix element on various planes of complex momenta. However, not all the poles and zeroes are equally important. One can see that below the $K\overline{K}$ threshold only a pole on sheet $(-++)$ and a zero on sheet $(+++)$ lie near physical region in all the complex momenta planes. Therefore, they play the most important role in that region and can be related to resonances $f_0(500)$ (or σ) and $f_0(980)$. The second resonance is still interpreted either as an ordinary $q\bar{q}$ state or as a $K\overline{K}$ quasi-bound state. It was already pointed out in [2] that the nature of $f_0(980)$ can be investigated by watching positions of the poles while the interchannel couplings go to zero. In two of our solutions we find that in the fully decoupled case the $f_0(980)$ poles are located on the positive part of imaginary k_K axis forming the $K\overline{K}$ bound state. In two other solutions the corresponding poles lie below the real k_K axis thus exhibiting the ordinary resonance nature of $f_0(980)$. In order to resolve this

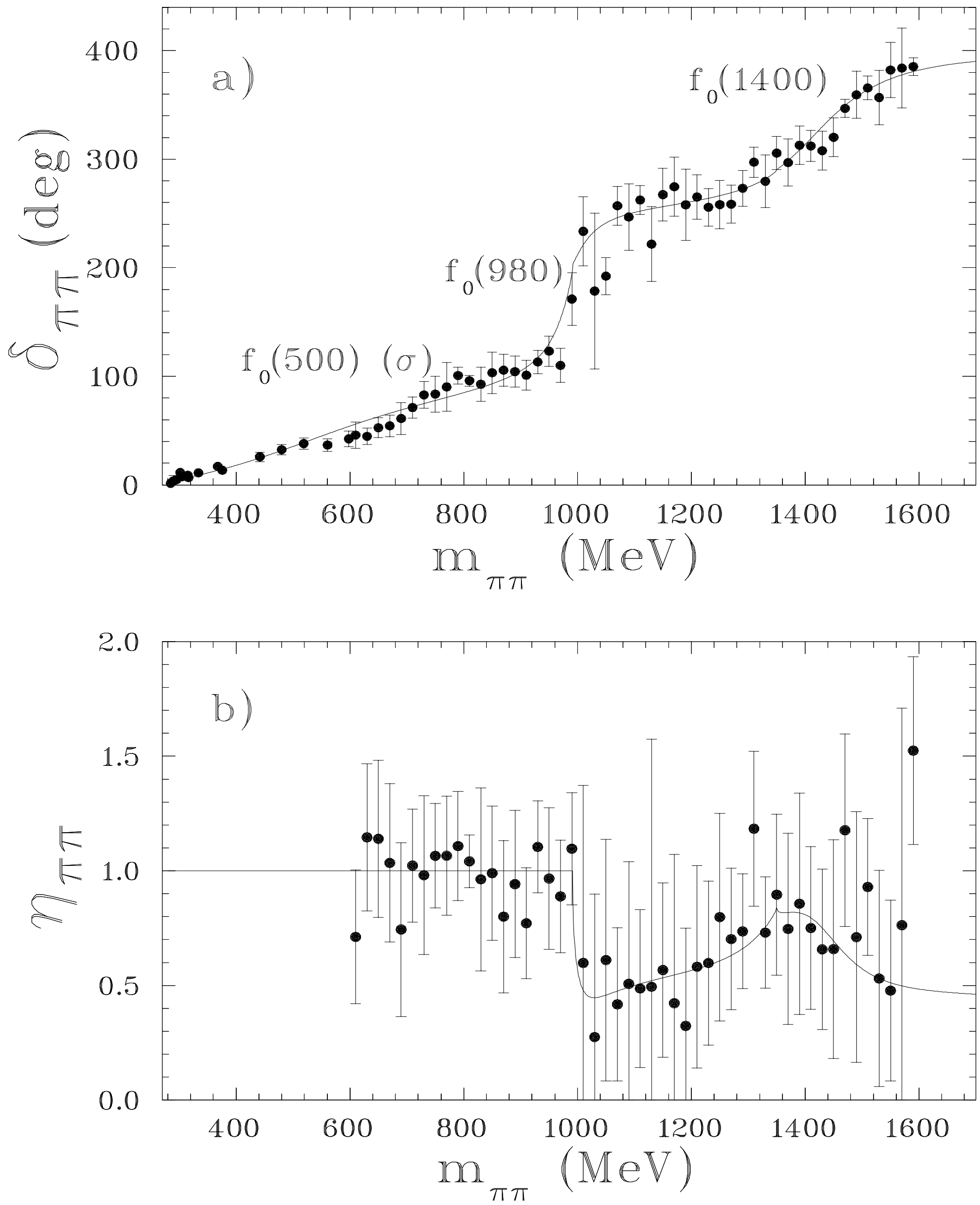

Figure 1: Energy dependence of **a)** $\pi\pi$ phase shifts and **b)** $\pi\pi$ inelasticity for the solution B of [2]. Experimental data above $m_{\pi\pi} = 600$ MeV correspond to the "down-flat" solution from [4].

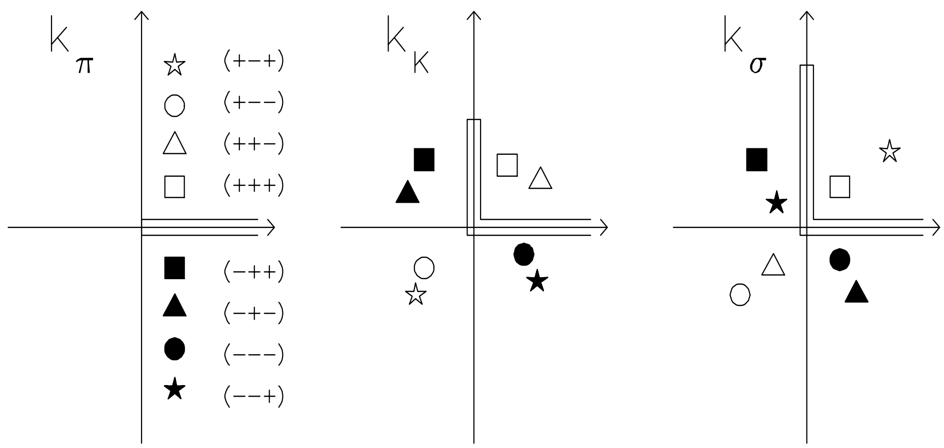

Figure 2: Complex momentum planes with schematic positions of the $S_{\pi\pi}$ zeroes (empty symbols) and poles (full symbols). Sheet positions are given on the k_π plane. For clarity, positions of the symmetric poles and zeroes with respect to the imaginary axes are not plotted. Double lines denote physical regions in the particular channels.

ambiguity one needs, however, new and very precise experimental data on the $K\overline{K}$ interaction amplitudes near threshold.

Near the $\sigma\sigma$ threshold we find four poles but in Table 1 only two of them are listed. The two poles on sheets $(- - -)$ and $(- - +)$ play a dominant role in the $\sigma\sigma$ channel. Different positions of those poles may lead to their interpretation as two distinct resonances (with different masses, widths, branching ratios etc.). We know, however, that in three of our solutions they come from the "bare" $\pi\pi$ resonance which splits into four states due to the interchannel couplings. In one case, namely in the solution B, two poles near 1400 MeV have their origin in the $\pi\pi$ channel and the other two in the $\sigma\sigma$ channel. Since the masses of resonances determined by positions of the poles are similar, in Table 1 we use one common name for them: $f_0(1400)$. Note that our two $f_0(1400)$ poles can be compared to the wide $f_0(1370)$ and the narrow $f_0(1500)$ listed in [1].

Other properties of resonances listed in Table 1 (e.g. branching ratios) together with a discussion of the limited applicability of the Breit-Wigner approach can be found in our paper [2].

Table 1: Average masses and widths of resonances $f_0(500)$, $f_0(980)$ and $f_0(1400)$ found in our solutions A, B, E and F (fits to "down-flat" data sets). Errors represent the maximum departure from the average.

resonance	mass (MeV)	width (MeV)	sheet
$f_0(500)$ or σ	523 ± 12	518 ± 14	$- + +$
$f_0(980)$	991 ± 3	71 ± 14	$- + +$
	1406 ± 19	160 ± 12	$- - -$
$f_0(1400)$	1447 ± 27	108 ± 46	$- - +$

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